

Corrections and Changes of “Lévy Processes and Infinitely Divisible Distributions”

Ken-iti Sato

Page a , line b from the top and page a , line c from the bottom are denoted by a^b and a_c , respectively. Page a , reference d is denoted by $a [d]$. “Replace ABC by XYZ ” is denoted by “ $ABC \rightarrow XYZ$ ”.

- xi₁₄ : Insert the following at the top of the line: $A^c = \mathbb{R}^d \setminus A$,
- xii⁸ : Insert the following at the end of the line:
See p. 3 for the meaning of $\{X_t\} \stackrel{d}{=} \{Y_t\}$.
- xii₈ : that is \rightarrow that is,
- xii₈ : Insert the following at the end of the line:
(However, the prime is sometimes used not in this way. For example, together with a stochastic process $\{X_t\}$ taking values in \mathbb{R}^d , we use $\{X'_t\}$ for another stochastic process taking values in \mathbb{R}^d ; X'_t is not the transpose of X_t .)
- xii₄ : in $\mathbb{R}^d \rightarrow$ on \mathbb{R}^d
- 5⁶ : special case of \rightarrow special case of a discrete version of
- 7₄ : bounded \rightarrow finite
- 11¹⁷ : $e \rightarrow e$
- 13₇ : It is exponential if $c = 1$. \rightarrow Sometimes c is called shape parameter and α scale parameter. If $c = 1$, then μ is exponential.
- 15⁶ : *mean* \rightarrow *parameter*
- 24⁸ : $P \rightarrow E$
- 46₄ : Replace the period (.) at the end of the line by a semicolon (;)
- 46₁ : Delete the semicolon (;) at the end of the line.
- 47¹ : Add the following at the top of the line:
(Weibull with $\alpha > 1$ is not infinitely divisible, see Exercise 29.10);
- 47¹⁵ and 47¹⁶ : $dx \rightarrow dx$ (each two places)
- 49₁₅ : $(a_{nk}) \rightarrow (a_{nk})_j$
- 52₈ : $A_{s_n} \rightarrow A_{s_n} z$
- 52₅ : $A_{s_n} \rightarrow A_{s_n} z$

- 53³ : Lévy \longrightarrow Lévy–Khintchine
- Insert the following between 66₂ and 66₁ :
 - (3) $\lim_{l \rightarrow \infty} \sup_{\mu \in M} \int_{|x| > l} \nu_{\mu}(dx) = 0$;
- 66₁ : (3) \longrightarrow (4)
- 68₂₂ : [201, 204] \longrightarrow [199, 201, 204]
- 68₂₀ : method. \longrightarrow method. Itô [204] also explains this method.
- 70₁₄ : distribution with \longrightarrow distribution μ with
- 70₁₃ : stable. The semi-stability \longrightarrow stable, as
- 72₆ : $Z \longrightarrow W$
- 77₁₅ : $\{|x| = 1\}$, the unit sphere, and let, for $b > 1$, $\longrightarrow \{|x| = 1\}$.
It is the unit sphere (if $d \geq 2$) or $\{1, -1\}$ (if $d = 1$). For $b > 1$, let
- 79₁₆ : Make clear the printing of the second “ $<$ ” in “ $0 < \alpha < 2$ ”
- 84₃ : of (14.18) gives \longrightarrow and (14.18) give
- 89⁸ : Insert the following above this line:
Notice that $\sin \pi \rho \alpha > 0$ if $\alpha \neq 0$ and $\beta \neq -1$.
- 90⁵ : $(\frac{c}{c_1} - i\tau \frac{c}{c_1} \operatorname{sgn} z) \longrightarrow (\frac{c}{c_1} - i\tau \frac{1}{c_1} \operatorname{sgn} z)$
- 90₁ : Add the following at the end of the line:
(Remarks concerning the definition will be given in Proposition 15.5 and Exercise 18.14.)
- 95¹⁴ : unit sphere. \longrightarrow unit sphere (if $d \geq 2$) or the two-point set $\{1, -1\}$ (if $d = 1$).
- 95₁₃ : Add the following at the top of the line:
Since $\int_0^{\infty} 1_B(r\xi) k_{\xi}(r) \frac{dr}{r} = \int_0^{\infty} 1_B(r\xi) k_{\xi}(r+) \frac{dr}{r}$, it is measurable in ξ .
- 97⁷ : ν [2 places] $\longrightarrow \mu$
- 97¹⁹ : $c(\xi) > 0 \longrightarrow c(\xi)$ with $0 < \xi < \infty$
- 97²² : $k_{\xi}^{\#}(\cdot) = c(\xi)^{-1} k_{\xi}(\cdot) \longrightarrow k_{\xi}^{\#}(r) dr = c(\xi)^{-1} k_{\xi}(r) dr$
- 97₉ : $\nu(d\xi) \longrightarrow \nu(dx)$
- 97₅ : Add the following at the end of the line:
See Lemma 59.3 for another uniqueness condition.
- 97₃ : 1 and $\longrightarrow 1$,
- 97₃ : of ξ . \longrightarrow of ξ , and $k_{\xi}(r)$ is right-continuous in $r > 0$.

- 97₁ : Add the following at the end of the line:

Recall that a decreasing function has a countable number of jumps at most and hence coincides with its right-continuous modification except at a countable number of points.

- 99₃ : selfsimilar process \longrightarrow selfsimilar additive process
- 105₅ : $e^{i\langle a_j z, Z_{u_j} - Z_{u_{j-1}} \rangle} \longrightarrow E \left[e^{i\langle a_j z, Z_{u_j} - Z_{u_{j-1}} \rangle} \right]$

- 108¹¹ : Add the following at the end of the line:

This process is sometimes called an *Ornstein–Uhlenbeck process driven by a Lévy process*.

- 109⁹ : unit sphere \longrightarrow unit sphere (if $d \geq 2$) or $\{1, -1\}$ (if $d = 1$),
- 109¹¹ : Add the following at the top of the line:

Sometimes we say that ρ has finite log-moment if (17.11) is satisfied.

- 115₁ : $c \neq 0 \longrightarrow c = 0$
- 116₁₄ : $dx \longrightarrow dx$
- 116₁₃ : (2 places) $dx \longrightarrow dx$
- 116₁₂ – 116₁₀ : (By scaling and translation we can make $a = c = 1$. The case $a = c = 1$ appears as the distribution of $Y = \log X$, where X has Γ -distribution with parameter $b, 1$.)

\longrightarrow (If X has Γ -distribution with parameters $a^{-1}b, c$, then $a^{-1} \log X$ has distribution $f(x)dx$.)

- 119¹⁰ : simultaneously subtracted means \longrightarrow means simultaneously subtracted
- 120¹³ : (1.6) \longrightarrow 1.6
- 120₁₄ : $d(s, x) \longrightarrow d(s, x)$
- 121₁₈ : Move “Also” to the top of the line.
- 125₁₅ : $\varepsilon \downarrow \infty \longrightarrow \varepsilon \downarrow 0$
- 127₄ : is 0 \longrightarrow tends to 0
- 146¹⁷ : $\dim \mu \longrightarrow \dim_{\mathbb{R}} \mu$
- 152¹² : Insert a period (.) between $[t\gamma_0, \infty)$ and *If*

- 157¹⁰ : Replace this line by the following:
 $\{X_t\}$ is degenerate. If $\{X_t\}$ is degenerate, then it is possible that $\{X_t\}$ is genuinely d -dimensional.
- 160₆–160₅ : Lévy representation
 \longrightarrow Lévy–Khintchine representation
- 161₇ : $g(x + y) \leq abe^{c|x|}g(x + y)$ by Lemma 25.5, \longrightarrow $g(x + y)$,
- 163⁹ – 163¹⁰ : although it has support \mathbb{R} for every $t > 0$ (Theorem 24.10(i)).
 \longrightarrow (it has support \mathbb{R} for every $t > 0$ as is shown in Theorem 24.10(i)).
- 163₉ : Embrecht \longrightarrow Embrechts
- 164⁶ : $[X_1 \leq x, \dots, X_{j-1} \leq x, X_j > x]$
 \longrightarrow $[Z_1 \leq x, \dots, Z_{j-1} \leq x, Z_j > x]$
- 164⁸ : $\{X_t\}$ be \longrightarrow $\{X_t\}$ is
- 164₁₈ : $L(x) \neq 0$ \longrightarrow $L(x)$ is positive, measurable,
- 164₅ : Embrecht \longrightarrow Embrechts
- 166⁹ : P_{X_t} \longrightarrow $P_{X_1(t)}$
- 178₈ : $x_1 \cdots + x_d$ \longrightarrow $x_1 + \cdots + x_d$
- 181₁₅ : Add the following at the end of the line:
 See Exercise 29.13 for an extension to non-infinitely-divisible case.
- 182⁵ : $c = 1$ \longrightarrow $c_1 = 1$
- 190₅ : $|\tilde{\mu}(z)|$ \longrightarrow $|\hat{\mu}(z)|$
- 191₁₅ : $z > b$ \longrightarrow $z > 1/b$
- 196 : Replace the four lines 196₆–196₃ by the following:
 to $[0, 1] \cup \{\infty\}$, Rubin [384] describes the construction of a Lévy process $\{X_t\}$ on \mathbb{R} such that $\dim_{\mathbb{R}} P_{X_t} = f(t)$. Here, for any singular distribution μ , $\dim_{\mathbb{R}} \mu$ is equal to the Hausdorff dimension $\dim_{\mathbb{H}} \mu$ defined as the infimum of the Hausdorff dimensions of all Borel sets B with $\mu(B) = 1$. If μ is not singular, then $\dim_{\mathbb{R}} \mu$ is defined to be ∞ .
- 199¹–199² : We have
 \longrightarrow Noting that $\{X_2(t)\}$ is a compound Poisson process, we have

- 202³ : $Y_{Z_2(t)} \longrightarrow Y'_{Z_2(t)}$
- 212₁₀ : given \longrightarrow made explicit
- 217₁₆ : In this way the logarithm of an operator is defined.
 \longrightarrow This is a way to define the logarithm of an operator.
- 220₄ : satisfying (33.3) \longrightarrow satisfying (33.3) and $-\infty < \varphi(x) < \infty$
- 221¹² : Delete “*positive*”.
- 230₁₇ : $P^\# \longrightarrow E^{P^\#}$
- 236₅ : Insert “Newman [324] and” before “Brockett”.
- 236₄ : Delete “ $A = A^\#$ and”.
- 240¹ : Insert “*Fix* $a > 0$.” before the first sentence of LEMMA 35.5.
- 240³ : $a > 0 \longrightarrow \varepsilon > 0$
- 241¹⁸ : if \longrightarrow If
- 250¹⁶ : Add the following at the end of line:
This remark continues to Remark 37.13.
- 253₂ : $\leq 0 \longrightarrow \geq 0$
- 256 : The last two lines should be as follows:

$$K^+ = \int_{(2,\infty)} x \left(\int_{-x}^{-1} \nu(-\infty, y) dy \right)^{-1} \nu(dx),$$

$$K^- = \int_{(-\infty,-2)} |x| \left(\int_1^{|x|} \nu(y, \infty) dy \right)^{-1} \nu(dx).$$

- 257 : Insert the following between 257⁴ and 257⁵:
See [115], p.373, where these are proved by the reduction to the results on random walks.
- 257⁵ : Begin a new paragraph.
- 257⁶ : Add the following at the end of the line:
Indeed this is obvious if ‘are respectively equivalent to’ is replaced by ‘respectively imply’; then note that (1), (2), and (3) are exhaustive. We now have a criterion of drifting to ∞ , drifting to $-\infty$, and oscillating for Lévy processes on \mathbb{R} in terms of Lévy measure and parameter γ .
- 270 : Make the vertical space between 270¹⁰ and 270¹¹ shorter.
- 276₆ : $\mathcal{F}_{t-s} \longrightarrow \mathcal{F}_{(t-s) \vee 0}$
- 276₂ : $\bigcup_k \longrightarrow \bigcap_k$

- 281⁹ : $t \geq 0 \longrightarrow s \geq 0$
- 281⁹ : $\Omega' \longrightarrow \Omega' \cap \{X_0 = 0\}$
- 284¹⁴ : $X_t \longrightarrow X_T$
- 285¹⁷ : $\int_0^\infty e^{-t-rt/q} P_{t/q} f dt \longrightarrow \int_0^\infty e^{-t-qt/r} P_{t/r} f dt$
- 287¹⁰ : $X_t \longrightarrow X_s$
- 288₅ : $f_n(x) \longrightarrow f_n(y)$
- 301₇ : $C(B) \longrightarrow C^q(B)$
- 305₁₂ : (7) \longrightarrow (1)
- 305₁₁ : (8) \longrightarrow (2)
- 305₁₀ : (9) \longrightarrow (3)
- 305₇ : (10) \longrightarrow (4)
- 307¹¹ : *Proof of* \longrightarrow *Proof of*
- 313¹⁶ : Move “the set” to the top of the line.
- 327₁₆ : $1_{\mathbb{R} \setminus \{0\}} \longrightarrow 1_{\mathbb{R}^d \setminus \{0\}}$
- 328¹⁴ : $H(x, t, \omega) \longrightarrow H(y, t, \omega)$
- 328¹⁶ : $H(x, t) \longrightarrow H(y, t)$
- 337₅ : right-hand sides \longrightarrow left-hand sides
- 338² : Replace the period (.) at the end of the line by a comma (,)
- 343⁸ : $\nu(dx) \longrightarrow x\nu(dx)$
- 350₅ : $> 0 \longrightarrow \geq 0$
- 358⁹ : [114] \longrightarrow [113]
- 359³ : $e \longrightarrow e$
- 359₁₄ : Proposition 47.14.
 \longrightarrow Proposition 47.14 for $t \rightarrow \infty$ instead of $t \downarrow 0$.
- 368₁ : [113] \longrightarrow [114]
- 377₉ : $\int_t^\infty \longrightarrow \int_1^\infty$
- 378¹⁷ : symmetric \longrightarrow symmetric with $A = 0$
- 381¹³ : $\log \log(1/s) \longrightarrow \log \log(1/u)$
- 381¹⁴ : $0 < s \longrightarrow 0 < u$

- 384¹² : The displayed equation should be as follows:

$$E[e^{-uL^{-1}(1)-vM(L^{-1}(1))}] = \exp\left[-c \exp\left[\int_0^\infty t^{-1} dt \int_{(0,\infty)} (e^{-t} - e^{-ut-vx}) \mu^t(dx)\right]\right]$$

- 388⁹ : Theorem 51.3 shows that μ_n is infinitely divisible. Hence,
 - By Theorem 51.3 μ_n is infinitely divisible and
- 388₁₆ : $(0, \infty) \longrightarrow [0, \infty)$
- 388₁₄ : $(0, \infty) \longrightarrow [0, \infty)$
- 388₁₀ : Add the following at the end of the line:

Note that $\rho(\{0\}) = \lim_{x \rightarrow \infty} f(x)$.
- 389¹⁵ : the Bondesson class
 - the Bondesson class or the Goldie–Steutel–Bondesson class, because it is related to Goldie [151], Steutel [441], and Bondesson [46]
- 393₄ : Make clear the printing of “e” in “mixture”
- 404¹⁵ : $Ke^{c-1} \longrightarrow Kx^{c-1}$
- 424⁴ : Replace the period (.) by a comma (,)
- 424⁵ : Delete this line.
- 425₁₃ : Add the following at the end of the line:

(A distribution in T is called generalized gamma convolution or GGC.)
- 429³ : (3a) \longrightarrow (4a)
- 429⁴ : (1), (2), and (3) \longrightarrow (1), (2), (3), and (4)
- 429⁴ : (1), (2), and (3a) \longrightarrow (1), (2), (3), and (4a)
- 430₇ : Add the following at the end of line:

See E 18.18 for another example.
- 433¹³ – 433¹⁴ : in the case $a = b = c = 1$ follows also from E 29.16. See also Theorem 2 of [419].
 - is evident from the expression of the Lévy measure; see Theorem 2 of [419] for another proof. It also follows from E 29.16 if $a^{-1}b = 1$ and $c = 1$.
- 434¹ : $X_n \longrightarrow S_n$
- 437¹⁷ : $x_k^{-1} \longrightarrow p_k x_k^{-1}$
- 437¹⁷ : $|x_{-l}|^{-1} \longrightarrow p_l |x_{-l}|^{-1}$
- 439¹³ : Use E 34.3 \longrightarrow Use the result of [337], p. 159,
- 444¹⁹ : 406–407 \longrightarrow 425–426

- 456 [109] : Embrecht \longrightarrow Embrechts
- 456 [110] : Embrecht \longrightarrow Embrechts
- 456²⁵–456²⁸ should be as follows:
 - [113] Erdős, P. (1942) On the law of the iterated logarithm, *Ann. Math.* **43**, 419–436. 358
 - [114] Erdős, P. and Révész, P. (1997) On the radius of the largest ball left empty by a Wiener process, *Stud. Sci. Math. Hungar.* **33**, 117–125. 368
- 459 [175] : (1973) \longrightarrow (1972)
- 460 [202] : Replace the two lines by the following:
 - [202] Itô, K. (2006) *Essentials of Stochastic Processes*, Amer. Math. Soc., Providence, RI. [Japanese original 1957] 68,236
- 460 [204] : Replace the two lines by the following:
 - [204] Itô, K. (2004) *Stochastic Processes. Lectures Given at Aarhus University* (ed. O. Barndorff-Nielsen and K. Sato), Springer, Berlin. [Original lecture notes 1969] 30,68₂,196₂
- 464₅, 464₂, 465², 465⁶, 465¹⁰ : Gauthie \longrightarrow Gauthier
- 465 [300] : (1998) \longrightarrow (1999)
- 465 [300] : *Probab.*, to appear. \longrightarrow *Probab.* **12**, 347–373.
- 466 [322] : reversal \longrightarrow reversion
- 466₁ : 236₂ \longrightarrow 236₃
- 467 [333] : Replace the two lines by the following:
 - [333] Petrov, V. V. (1975) *Sums of Independent Random Variables*, Springer, Berlin. [Russian original 1972] 196
- 467 [336] : Some stable \longrightarrow Semi stable
- 467 [337] : 234 \longrightarrow 234,439
- 469²⁷ : *see also* [113] \longrightarrow *see also* [114]
- 469 [376] : (1994) \longrightarrow (1999)
- 469 [376] : 2nd ed. \longrightarrow 3rd ed.
- 470 [397] : 118, \longrightarrow 118
- 470 [398] : *Probabability* \longrightarrow *Probability*
- 471 [408] : to appear. \longrightarrow 129–145.

- 473 [440] : Notes \longrightarrow Note
- 474 [462] : waks \longrightarrow walks
- 474 [469] : ststationary \longrightarrow stationary
- 474₄ : Delete the period (.) at the end of the line.
- 476 [496] : (1998) \longrightarrow (2000)
- 476 [496] : Preprint.
 \longrightarrow *Prob. Theory Related Fields* **117**, 387–405.
- 476 [497] : *Japan. J. Math.*, to appear.
 \longrightarrow *Japan. J. Math.* **25**, 227–256.
- 477 [516] : (1998) \longrightarrow (2000)
- 477 [516] : *J. Math. Soc. Japan*, to appear.
 \longrightarrow *J. Math. Soc. Japan* **52**, 343–362.
- 478 [534] : 653–664 \longrightarrow 653–662
- Erase the irregular dots that exist in the following places:
17³, 27₅, 39₃, 51₇, 59₆, 68 (foot margin), 99₆, 123₄, 131₇, 146⁵, 147₇, 148³, 155₈,
187₇, 195₈, 199¹⁵, 203₇, 218¹⁷, 220¹⁴, 231₂, 231₁, 248₁₃, 248₁₂, 260 (between
260₉ and 260₈), 261 (foot margin), 263₈ (two dots), 292¹¹, 314¹⁴, 323 (between
323₆ and 323₅), 326₁, 327 (between 327¹ and 327²), 331₉ (below “l” of “total”),
342⁴ (between lines 342³ and 342⁵), 342⁶, 347 (between 347₄ and 347₃), 350₁₄,
363₇, 379₇, 382 (a blur in foot margin), 386⁷, 395₇, 398², 403₈, 404 (between
404₁₃ and 404₁₂), 408⁶ (above “af”), 419₇, 423 (foot margin), 437 (between
437¹² and 437¹³), 443 (between 443₁₀ and 443₉), 457 [134], 482₂₁ (left column)
(left margin).

(May 30, 2013)