Corrections and Changes of
"Lévy Processes and Infinitely Divisible Distributions"

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Page a, line b from the top and page a, line c from the bottom are denoted by a^b and a_c, respectively. Page a, reference d is denoted by a $[d]$. "Replace ABC by XYZ" is denoted by "$ABC \quad \longrightarrow \quad XYZ$".

- xi_{14} : Insert the following at the top of the line: $\quad A^c = \mathbb{R}^d \setminus A$,
- xii^8 : Insert the following at the end of the line:
  See p. 3 for the meaning of $\{X_t\} \stackrel{\mathrm{d}}{=} \{Y_t\}$.
- xii_8 : that is $\longrightarrow$ that is,
- xii_8 : Insert the following at the end of the line:
  (However, the prime is sometimes used not in this way. For example, together with a stochastic process $\{X_t\}$ taking values in $\mathbb{R}^d$, we use $\{X'_t\}$ for another stochastic process taking values in $\mathbb{R}^d$; X'_t is not the transpose of X_t.)
- xii_4 : in $\mathbb{R}^d$ $\longrightarrow$ on $\mathbb{R}^d$
- 5^6 : special case of $\longrightarrow$ special case of a discrete version of
- 7_4 : bounded $\longrightarrow$ finite
- 11^{17} : e $\longrightarrow$ e
- 13_7 : It is exponential if $c = 1$. $\longrightarrow$ Sometimes c is called shape parameter and α scale parameter. If $c = 1$, then μ is exponential.
- 15^6 : *mean* $\longrightarrow$ *parameter*
- 24^8 : P $\longrightarrow$ E
- 46_4 : Replace the period (.) at the end of the line by a semicolon (;)
- 46_1 : Delete the semicolon (;) at the end of the line.
- 47^1 : Add the following at the top of the line:
  (Weibull with $\alpha > 1$ is not infinitely divisible, see Exercise 29.10);
- 47^{15} and 47^{16} : dx $\longrightarrow$ $\mathrm{d}x$ (each two places)
- 49_{15} : (a_{nk}) $\longrightarrow$ $(a_{nk})_j$
- 52_8 : A_{s_n} $\longrightarrow$ $A_{s_n}z$
- 52_5 : A_{s_n} $\longrightarrow$ $A_{s_n}z$

- 53^3 : Lévy $\longrightarrow$ Lévy–Khintchine
- Insert the following between 66_2 and 66_1 :

  (3) $\lim_{l\to\infty} \sup_{\mu\in M} \int_{|x|>l} \nu_\mu(\mathrm{d}x) = 0$;
- 66_1 : (3) $\longrightarrow$ (4)
- 68_{22} : [**201, 204**] $\longrightarrow$ [**199, 201, 204**]
- 68_{20} : method. $\longrightarrow$ method. Itô [**204**] also explains this method.
- 70_{14} : distribution with $\longrightarrow$ distribution μ with
- 70_{13} : stable. The semi-stability $\longrightarrow$ stable, as
- 72_6 : Z $\longrightarrow$ W
- 77_{15} : $|x| = 1\}$, the unit sphere, and let, for $b > 1$, $\longrightarrow$ $|x| = 1\}$. It is the unit sphere (if $d \geq 2$) or $\{1, -1\}$ (if $d = 1$). For $b > 1$, let
- 79_{16} : Make clear the printing of the second "$<$ " in "$0 < \alpha < 2$ "
- 84_3 : of (14.18) gives $\longrightarrow$ and (14.18) give
- 89^8 : Insert the following above this line:

  Notice that $\sin \pi\rho\alpha > 0$ if $\alpha \neq 0$ and $\beta \neq -1$.
- 90^5 : $(\frac{c}{c_1} - \mathrm{i}\tau\frac{c}{c_1}\mathrm{sgn}\, z)$ $\longrightarrow$ $(\frac{c}{c_1} - \mathrm{i}\tau\frac{1}{c_1}\mathrm{sgn}\, z)$
- 90_1 : Add the following at the end of the line:

  (Remarks concerning the definition will be given in Proposition 15.5 and Exercise 18.14.)
- 95^{14} : unit sphere. $\longrightarrow$ unit sphere (if $d \geq 2$) or the two-point set $\{1, -1\}$ (if $d = 1$).
- 95_{13} : Add the following at the top of the line:

  Since $\int_0^\infty 1_B(r\xi)k_\xi(r)\frac{\mathrm{d}r}{r} = \int_0^\infty 1_B(r\xi)k_\xi(r+)\frac{\mathrm{d}r}{r}$, it is measurable in ξ.
- 97^7 : ν [2 places] $\longrightarrow$ μ
- 97^{19} : $c(\xi) > 0$ $\longrightarrow$ $c(\xi)$ with $0 < \xi < \infty$
- 97^{22} : $k^\sharp_\xi(\cdot) = c(\xi)^{-1}k_\xi(\cdot)$ $\longrightarrow$ $k^\sharp_\xi(r)\mathrm{d}r = c(\xi)^{-1}k_\xi(r)\mathrm{d}r$
- 97_9 : $\nu(\mathrm{d}\xi)$ $\longrightarrow$ $\nu(\mathrm{d}x)$
- 97_5 : Add the following at the end of the line:

  See Lemma 59.3 for another uniqueness condition.
- 97_3 : 1 and $\longrightarrow$ 1,
- 97_3 : of ξ. $\longrightarrow$ of ξ, and $k_\xi(r)$ is right-continuous in $r > 0$.

- 97_1 : Add the following at the end of the line:

  Recall that a decreasing function has a countable number of jumps at most and hence coincides with its right-continuous modification except at a countable number of points.
- 99_3 : selfsimilar process $\longrightarrow$ selfsimilar additive process
- 105_5 : $\mathrm{e}^{\mathrm{i}\langle a_j z, Z_{u_j} - Z_{u_{j-1}}\rangle}$ $\longrightarrow$ $E\left[\mathrm{e}^{\mathrm{i}\langle a_j z, Z_{u_j} - Z_{u_{j-1}}\rangle}\right]$
- 108^{11} : Add the following at the end of the line:

  This process is sometimes called an *Ornstein–Uhlenbeck process driven by a Lévy process.*
- 109^9 : unit sphere $\longrightarrow$ unit sphere (if $d \geq 2$) or $\{1, -1\}$ (if $d = 1$),
- 109^{11} : Add the following at the top of the line:

  Sometimes we say that ρ has finite log-moment if (17.11) is satisfied.
- 115_1 : $c \neq 0$ $\longrightarrow$ $c = 0$
- 116_{14} : dx $\longrightarrow$ $\mathrm{d}x$
- 116_{13} : (2 places) dx $\longrightarrow$ $\mathrm{d}x$
- 116_{12} – 116_{10} : (By scaling and translation we can make $a = c = 1$. The case $a = c = 1$ appears as the distribution of $Y = \log X$, where X has Γ-distribution with parameter b, 1.)

  $\longrightarrow$ (If X has Γ-distribution with parameters $a^{-1}b$, c, then $a^{-1}\log X$ has distribution $f(x)\mathrm{d}x$.)
- 119^{10} : simultaneously subtracted means

  $\longrightarrow$ means simultaneously subtracted
- 120^{13} : *(1.6)* $\longrightarrow$ *1.6*
- 120_{14} : $d(s, x)$ $\longrightarrow$ $\mathrm{d}(s, x)$
- 121_{18} : Move “Also” to the top of the line.
- 125_{15} : $\varepsilon \downarrow \infty$ $\longrightarrow$ $\varepsilon \downarrow 0$
- 127_4 : is 0 $\longrightarrow$ tends to 0
- 146^{17} : $\dim \mu$ $\longrightarrow$ $\dim_{\mathrm{R}} \mu$
- 152^{12} : Insert a period (.) between $[t\gamma_0, \infty)$ and *If*

- 157^{10} : Replace this line by the following:

  $\{X_t\}$ is degenerate. If $\{X_t\}$ is degenerate, then it is possible that $\{X_t\}$ is genuinely d-dimensional.
- 160_6–160_5 : Lévy representation $\longrightarrow$ Lévy–Khintchine representation
- 161_7 : $g(x+y) \le abe^{c|x|}g(x+y)$ by Lemma 25.5, $\longrightarrow$ $g(x+y)$,
- 163^9 – 163^{10} : although it has support $\mathbb{R}$ for every $t > 0$ (Theorem 24.10(i)).

  $\longrightarrow$ (it has support $\mathbb{R}$ for every $t > 0$ as is shown in Theorem 24.10(i)).
- 163_9 : Embrecht $\longrightarrow$ Embrechts
- 164^6 : $[X_1 \le x, \ldots, X_{j-1} \le x,\ X_j > x]$

  $\longrightarrow$ $[Z_1 \le x, \ldots, Z_{j-1} \le x,\ Z_j > x]$
- 164^8 : $\{X_t\}$ be $\longrightarrow$ $\{X_t\}$ is
- 164_{18} : $L(x) \neq 0$ $\longrightarrow$ $L(x)$ is positive, measurable,
- 164_5 : Embrecht $\longrightarrow$ Embrechts
- 166^9 : P_{X_t} $\longrightarrow$ $P_{X_1(t)}$
- 178_8 : $x_1 \cdots + x_d$ $\longrightarrow$ $x_1 + \cdots + x_d$
- 181_{15} : Add the following at the end of the line:

  See Exercise 29.13 for an extension to non-infinitely-divisible case.
- 182^5 : $c = 1$ $\longrightarrow$ $c_1 = 1$
- 190_5 : $|\widetilde{\mu}(z)|$ $\longrightarrow$ $|\widehat{\mu}(z)|$
- 191_{15} : $z > b$ $\longrightarrow$ $z > 1/b$
- 196 : Replace the four lines 196_6–196_3 by the following:

  to $[0,1] \cup \{\infty\}$, Rubin [**384**] describes the construction of a Lévy process $\{X_t\}$ on $\mathbb{R}$ such that $\dim_{\mathrm{R}} P_{X_t} = f(t)$. Here, for any singular distribution μ, $\dim_{\mathrm{R}} \mu$ is equal to the Hausdorff dimension $\dim_{\mathrm{H}} \mu$ defined as the infimum of the Hausdorff dimensions of all Borel sets B with $\mu(B) = 1$. If μ is not singular, then $\dim_{\mathrm{R}} \mu$ is defined to be ∞.
- 199^1–199^2 : We have

  $\longrightarrow$ Noting that $\{X_2(t)\}$ is a compound Poisson process, we have

- 202^3 : $Y_{Z_2(t)}$ $\longrightarrow$ $Y'_{Z_2(t)}$
- 212_{10} : given $\longrightarrow$ made explicit
- 217_{16} : In this way the logarithm of an operator is defined.

  $\longrightarrow$ This is a way to define the logarithm of an operator.
- 220_4 : satisfying (33.3) $\longrightarrow$ satisfying (33.3) and $-\infty < \varphi(x) < \infty$
- 221^{12} : Delete "*positive*".
- 230_{17} : $P^\sharp$ $\longrightarrow$ $E^{P^\sharp}$
- 236_5 : Insert "Newman [**324**] and" before "Brockett".
- 236_4 : Delete "$A = A^\sharp$ and".
- 240^1 : Insert "*Fix* $a > 0$." before the first sentence of LEMMA 35.5.
- 240^3 : $a > 0$ $\longrightarrow$ $\varepsilon > 0$
- 241^{18} : if $\longrightarrow$ If
- 250^{16} : Add the following at the end of line:

  This remark continues to Remark 37.13.
- 253_2 : ≤ 0 $\longrightarrow$ ≥ 0
- 256 : The last two lines should be as follows:

$$K^+ = \int_{(2,\infty)} x \left(\int_{-x}^{-1} \nu(-\infty, y) \mathrm{d}y \right)^{-1} \nu(\mathrm{d}x),$$
$$K^- = \int_{(-\infty,-2)} |x| \left(\int_1^{|x|} \nu(y, \infty) \mathrm{d}y \right)^{-1} \nu(\mathrm{d}x).$$

- 257 : Insert the following between 257^4 and 257^5:

  See [**115**], p. 373, where these are proved by the reduction to the results on random walks.
- 257^5 : Begin a new paragraph.
- 257^6 : Add the following at the end of the line:

  Indeed this is obvious if 'are respectively equivalent to' is replaced by 'respectively imply'; then note that (1), (2), and (3) are exhaustive. We now have a criterion of drifting to ∞, drifting to $-\infty$, and oscillating for Lévy processes on $\mathbb{R}$ in terms of Lévy measure and parameter γ.
- 270 : Make the vertical space between 270^{10} and 270^{11} shorter.
- 276_6 : $\mathcal{F}_{t-s}$ $\longrightarrow$ $\mathcal{F}_{(t-s)\vee 0}$
- 276_2 : $\bigcup_k$ $\longrightarrow$ $\bigcap_k$

- 281^9 : $t \geq 0 \longrightarrow s \geq 0$
- 281^9 : $\Omega' \longrightarrow \Omega' \cap \{X_0 = 0\}$
- 284^{14} : $X_t \longrightarrow X_T$
- 285^{17} : $\displaystyle\int_0^\infty \mathrm{e}^{-t-rt/q} P_{t/q} f \mathrm{d}t \longrightarrow \int_0^\infty \mathrm{e}^{-t-qt/r} P_{t/r} f \mathrm{d}t$
- 287^{10} : $X_t \longrightarrow X_s$
- 288_5 : $f_n(x) \longrightarrow f_n(y)$
- 301_7 : $C(B) \longrightarrow C^q(B)$
- 305_{12} : $(7) \longrightarrow (1)$
- 305_{11} : $(8) \longrightarrow (2)$
- 305_{10} : $(9) \longrightarrow (3)$
- 305_7 : $(10) \longrightarrow (4)$
- 307^{11} : *Proofof* $\longrightarrow$ *Proof of*
- 313^{16} : Move "the set" to the top of the line.
- 327_{16} : $1_{\mathbb{R}\setminus\{0\}} \longrightarrow 1_{\mathbb{R}^d\setminus\{0\}}$
- 328^{14} : $H(x,t,\omega) \longrightarrow H(y,t,\omega)$
- 328^{16} : $H(x,t) \longrightarrow H(y,t)$
- 337_5 : right-hand sides $\longrightarrow$ left-hand sides
- 338^2 : Replace the period (.) at the end of the line by a comma (,)
- 343^8 : $\nu(\mathrm{d}x) \longrightarrow x\nu(\mathrm{d}x)$
- 350_5 : $> 0 \longrightarrow \geq 0$
- 358^9 : [**114**] $\longrightarrow$ [**113**]
- 359^3 : $e \longrightarrow \mathrm{e}$
- 359_{14} : Proposition 47.14.

  $\longrightarrow$ Proposition 47.14 for $t \to \infty$ instead of $t \downarrow 0$.
- 368_1 : [**113**] $\longrightarrow$ [**114**]
- 377_9 : $\displaystyle\int_t^\infty \longrightarrow \int_1^\infty$
- 378^{17} : symmetric $\longrightarrow$ symmetric with $A = 0$
- 381^{13} : $\log\log(1/s) \longrightarrow \log\log(1/u)$
- 381^{14} : $0 < s \longrightarrow 0 < u$

- 384^{12} : The displayed equation should be as follows:

$$E[\mathrm{e}^{-uL^{-1}(1)-vM(L^{-1}(1))}] = \exp\big[-c\exp\big[\int_0^\infty t^{-1}\mathrm{d}t\int_{(0,\infty)}(\mathrm{e}^{-t}-\mathrm{e}^{-ut-vx})\mu^t(\mathrm{d}x)\big]\big]$$

- 388^{9} : Theorem 51.3 shows that μ_n is infinitely divisible. Hence,

  $\longrightarrow$ By Theorem 51.3 μ_n is infinitely divisible and
- 388_{16} : $(0,\infty)$ $\longrightarrow$ $[0,\infty)$
- 388_{14} : $(0,\infty)$ $\longrightarrow$ $[0,\infty)$
- 388_{10} : Add the following at the end of the line:

  Note that $\rho(\{0\}) = \lim_{x\to\infty} f(x)$.
- 389^{15} : the Bondesson class

  $\longrightarrow$ the Bondesson class or the Goldie–Steutel–Bondesson class, because it is related to Goldie [**151**], Steutel [**441**], and Bondesson [**46**]
- 393_{4} : Make clear the printing of "e" in "mixture"
- 404^{15} : $K\mathrm{e}^{c-1}$ $\longrightarrow$ Kx^{c-1}
- 424^{4} : Replace the period (.) by a comma (,)
- 424^{5} : Delete this line.
- 425_{13} : Add the following at the end of the line:

  (A distribution in T is called generalized gamma convolution or GGC.)
- 429^{3} : (3a) $\longrightarrow$ (4a)
- 429^{4} : (1), (2), and (3) $\longrightarrow$ (1), (2), (3), and (4)
- 429^{4} : (1), (2), and (3a) $\longrightarrow$ (1), (2), (3), and (4a)
- 430_{7} : Add the following at the end of line:

  See E 18.18 for another example.
- 433^{13} – 433^{14} : in the case $a=b=c=1$ follows also from E 29.16. See also Theorem 2 of [**419**].

  $\longrightarrow$ is evident from the expression of the Lévy measure; see Theorem 2 of [**419**] for another proof. It also follows from E 29.16 if $a^{-1}b=1$ and $c=1$.
- 434^{1} : X_n $\longrightarrow$ S_n
- 437^{17} : ${x_k}^{-1}$ $\longrightarrow$ $p_k\, {x_k}^{-1}$
- 437^{17} : $|x_{-l}|^{-1}$ $\longrightarrow$ $p_l\, |x_{-l}|^{-1}$
- 439^{13} : Use E 34.3 $\longrightarrow$ Use the result of [**337**], p. 159,
- 444^{19} : 406–407 $\longrightarrow$ 425–426

- 456 [109] : Embrecht $\longrightarrow$ Embrechts
- 456 [110] : Embrecht $\longrightarrow$ Embrechts
- 456^{25}–456^{28} should be as follows:

[113] Erdős, P. (1942) On the law of the iterated logarithm, *Ann. Math.* **43**, 419–436. *358*

[114] Erdős, P. and Révész, P. (1997) On the radius of the largest ball left empty by a Wiener process, *Stud. Sci. Math. Hungar.* **33**, 117–125. *368*

- 459 [175] : (1973) $\longrightarrow$ (1972)
- 460 [202] : Replace the two lines by the following:

[202] Itô, K. (2006) *Essentials of Stochastic Processes*, Amer. Math. Soc., Providence, RI. [Japanese original 1957] *68,236*

- 460 [204] : Replace the two lines by the following:

[204] Itô, K. (2004) *Stochastic Processes. Lectures Given at Aarhus University* (ed. O. Barndorff-Nielsen and K. Sato), Springer, Berlin. [Original lecture notes 1969] *30,*68_2*,*196_2

- 464_5, 464_2, 465^2, 465^6, 465^{10} : Gauthie $\longrightarrow$ Gauthier
- 465 [300] : (1998) $\longrightarrow$ (1999)
- 465 [300] : *Probab.*, to appear. $\longrightarrow$ *Probab.* **12**, 347–373.
- 466 [322] : reversal $\longrightarrow$ reversions
- 466_1 : 236_2 $\longrightarrow$ 236_3
- 467 [333] : Replace the two lines by the following:

[333] Petrov, V. V. (1975) *Sums of Independent Random Variables*, Springer, Berlin. [Russian original 1972] *196*

- 467 [336] : Some stable $\longrightarrow$ Semi stable
- 467 [337] : *234* $\longrightarrow$ *234,439*
- 469^{27} : *see also* [**113**] $\longrightarrow$ *see also* [**114**]
- 469 [376] : (1994) $\longrightarrow$ (1999)
- 469 [376] : 2nd ed. $\longrightarrow$ 3rd ed.
- 470 [397] : *118,* $\longrightarrow$ *118*
- 470 [398] : *Probabability* $\longrightarrow$ *Probability*
- 471 [408] : to appear. $\longrightarrow$ 129–145.

- 473 [440] : Notes $\longrightarrow$ Note
- 474 [462] : waks $\longrightarrow$ walks
- 474 [469] : ststionary $\longrightarrow$ stationary
- 474_4 : Delete the period (.) at the end of the line.
- 476 [496] : (1998) $\longrightarrow$ (2000)
- 476 [496] : Preprint.

  $\longrightarrow$ *Prob. Theory Related Fields* **117**, 387–405.
- 476 [497] : *Japan. J. Math.*, to appear.

  $\longrightarrow$ *Japan. J. Math.* **25**, 227–256.
- 477 [516] : (1998) $\longrightarrow$ (2000)
- 477 [516] : *J. Math. Soc. Japan*, to appear.

  $\longrightarrow$ *J. Math. Soc. Japan* **52**, 343–362.
- 478 [534] : 653–664 $\longrightarrow$ 653–662
- Erase the irregular dots that exist in the following places:

  17^3, 27_5, 39_3, 51_7, 59_6, 68 (foot margin), 99_6, 123_4, 131_7, 146^5, 147_7, 148^3, 155_8, 187_7, 195_8, 199^{15}, 203_7, 218^{17}, 220^{14}, 231_2, 231_1, 248_{13}, 248_{12}, 260 (between 260_9 and 260_8), 261 (foot margin), 263_8 (two dots), 292^{11}, 314^{14}, 323 (between 323_6 and 323_5), 326_1, 327 (between 327^1 and 327^2), 331_9 (below "l" of "total"), 342^4 (between lines 342^3 and 342^5), 342^6, 347 (between 347_4 and 347_3), 350_{14}, 363_7, 379_7, 382 (a blur in foot margin), 386^7, 395_7, 398^2, 403_8, 404 (between 404_{13} and 404_{12}), 408^6 (above "af"), 419_7, 423 (foot margin), 437 (between 437^{12} and 437^{13}), 443 (between 443_{10} and 443_9), 457 [134], 482_{21} (left column) (left margin).

(May 30, 2013)